EECS 861
Homework 9

1. Chapter 3: Problem 3.34 {Hint: find a function of T, g(T), then plot g(T) graphically find when g(T)< bound}

2. A zero mean WSS random process, X(t) has the following autocorrelation function
$$R_{XX}(\tau) = e^{-500|\tau|}$$
   a) Find E[X(t)]
   b) Find the variance of X(t).
   c) Find the effective bandwidth, B_{eff} of X(t).
   Let
$$Y = \frac{1}{T}\int_0^T X(\eta)d\eta$$
   d) For T = 1 ms find the variance of Y and the ratio of Var[X]/Var[Y].
   e) For T = 1000 ms find the variance of Y and the ratio of Var[X]/Var[Y]; then compare the Var[Y] to the large BT approximation of the of Var[Y].
   f) For T = 1000 ms find P(Y> 0.06)

3. A zero mean Gaussian WSS random process, X(t) has the following PSD
$$S_X(f) = \begin{cases} \frac{1}{10000} & |f| < 5000 \\ 0 & elsewhere \end{cases}$$
   a) Find E[X(t)]
   b) Find Var[X(t)]
   c) Plot $R_{XX}(\tau)$.
   d) Are the random variables X(t) and X(t-100μs) uncorrelated (Yes or No); justify?
   e) Are the random variables X(t) and X(t-100μs) statistically independent (Yes or No); justify?
   f) What is the bivariate pdf for the random variables X(t) and X(t-25μs).
   g) What is E[X(t)| X(t-400μs)=0]?

4. Given the random process given in problem 3 above what is the covariance matrix for the random variables X(t) and X(t-25μs)?

5. Given the random process given in problem 3 above find the variance of Y where
$$Y = \frac{1}{10}\sum_{k=1}^{10} X(t - k\Delta t) \quad \text{where } \Delta t = 200\mu s$$

6. Chapter 3: Problem 3.26

7. A WSS Gaussian random process X(t) has a PSD of $S_X(f) = \frac{40}{1+(4\pi f)^2}$

   Find
   a) $R_{XX}(\tau)$
   b) E[X(t)] and E[X(t-1)]
   c) Var[X(t)] and Var[X(t-1)]
   d) What is the covariance matrix for the random variables X(t) and X(t-1)?
   e) What is the correlation coefficient, $\rho_{X(t),X(t-1)}$?
   f) E[X(t)|X(t-1)=1]
   g) Var[X(t)|X(t-1)=1]
   h) P(X(t)>1|X(t-1)=1])

8. Let $R_{XX}(\tau) = \Lambda(\frac{\tau}{100})$
   a) Find $S_x(f)$
   b) Find B_{eff}
   c) Find the B_{3dB}
   d) Find $B_{\text{first zero}}$ defined as the first frequency where $S_x(f) = 0$
   e) Compare the above definitions of bandwidth.

9. Explain the difference between strict sense stationarity and ergodicity.

10. Let x(t) = sin2πt.
Assume x(t) is sampled and quantized with the following parameters:

   1) 4 uniformly spaced quantized levels
   2) sample rate = 5 samples/sec.
   3) sampling starts at t = 0
   4) assume impulse sampling

   a) What is the minimum number of bits per sample required?
   b) What is the output bit rate?
   c) Fill in the table shown below and show quantizer design.

sample number	sample value	quantized value	output code
1			
2			
3			
4			